Evaluate the integral and prove it converges: Wenzhi Tseng

The first equality follows from substituting . The second equality follows from the substitution and subsequently noting that . Recall that . To see converges, note is zero at and is positive for , so

So converges. Because is odd, contour integration is not so obvious here.

Instead, we use integration by parts,

Using L’Hopital’s, we have:

where the last equality follows from , which is itself proved using two

applications of L’Hopital’s. Then,

Note the power series converges on (can prove convergence at using alternating series test), hence it converges uniformly on , so we can interchange

integration and summation,

where denotes Catalan’s constant.